TIME DOMAIN SIMULATIONS OF A COUPLED PARAMETRICALLY EXCITED ROLL RESPONSE IN REGULAR AND IRREGULAR HEAD SEAS

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Abstract

In this study dynamic stability problems in head seas are investigated with the use of a time-domain, non-linear numerical model of ship's motions in 5 degrees-of-freedom (sway, heave, roll, pitch, and yaw). The results presented in this paper have shown that the ship can be subjected to the wave-induced parametric resonance in regular and also in irregular waves. A quasi-static approach is adopted to determine the non-linear restoring coefficients in heave, roll, and pitch motions in waves, in which calculations are made using a pressure integration technique over the instantaneous submerged hull. Dynamic and hydrodynamic effects in waves for a given encounter frequency are included in the 5 degrees-of-freedom responses calculations, which are based on the potential flow strip theory. Comparisons between computed values and applicable experimental results demonstrated the usefulness of the technique proposed.

1. INTRODUCTION

Since the early fifties [1], parametric resonance has been already been studied and discussed by several investigators and safety authorities. It has been found that a ship heading into a two-dimensional longitudinal wave system will experience periodic variations in her transverse stability, which under certain conditions will result in pronounced rolling. This phenomenom has been considered mostly of theoretical interest and less worthy of practical concern. However, recent evidence of parametric rolling in head seas on a post-Panamax C11 class container ship [2] received wide and renewed attention, demonstrating the practical importance of this phenomenon.

A monohull encountering waves with length nearly equal to the ship length will have significant variations on waterplane area relatively to still water condition. The righting arm decreases if a wave crest is amidships and increases when a trough is near amidships. In this dynamic situation, the ship motion should be described by coupled roll, pitch and heave, and hence the restoring forces and moments should include effects relevant to these motions.

In reference [3], it was demonstrated that both linearised and non-linear theories could be used to predict parametric rolling in regular head waves. On the linear model (in the form of Mathieu's equation) stability variations were evaluated from the linearised righting arm curves with the wave crest varying longitudinally along the ship hull. However, this model was not accurate enough to predict ship's roll response magnitude under wave-induced parametric resonance conditions, since deck submergence effect on restoring characteristics of the vessel could not be included and therefore the limit cycle behaviour could not be obtained.

A non-linear numerical model of parametric resonance taking into consideration deck submergence and other non-linearities on restoring moment of ships in regular waves was also proposed [3]. In that model a quasi-static approach was adopted to study the roll motion, where only the variations on transverse stability in regular waves were considered. For that purpose, an uncoupled roll equation, which included the effects of heave and pitch responses in regular waves, and immersed hull variations due to wave passage on roll restoring term, was used to describe the parametrically excited roll motions.

While good agreement in terms of limited response behaviour was found between the time domain simulation of roll motion in longitudinal regular waves and the existing experimental data, simulations of parametric rolling in irregular head waves as presented in literature by some authors [2, 4, 5] could not be performed using that 1 degree-of-freedom (dof) model.

To overcome these shortcomings a non-linear model, coupled in the 5 dof's has been developed to simulate the time domain responses of a ship in uni-directional long-crested irregular waves. Although this non-linear time domain simulation does not account for all non-linear hydrodynamic phenomena, the code is now more sophisticated and capable of predicting parametric rolling responses in a scenario more closely related to the ship's conditions that may be found at sea.

2. EQUATIONS OF MOTION

In this work unrestrained rigid body motions of a slender vessel with advancing speed are considered. The dynamics of oscillatory ship motions is governed by Newton's second law, which represent the equilibrium between the internal forces due to inertia, gravity, and the external forces acting on the ship, given by:

$$[M]\{\ddot{\xi}\} = [F] \qquad (1)$$

These forces $[F]$ and motions $\{\xi\}$ may be represented on a coordinate system (see Fig. 1) fixed with respect to the mean position of the ship, $X = (x, y, z)$, with z in the vertical upward direction and passing through the centre of gravity of the ship, x along the longitudinal direction of the ship and directed to the bow, and y perpendicular to the latter and in the port direction. The origin is in the plane of the undisturbed free surface. The translatory displacements in the x, y, and z directions are respectively the surge ξ_1, the sway ξ_2, and the heave ξ_3. The rotational displacements about the x, y, and z axes are respectively the roll ξ_4, the pitch ξ_5, and the yaw ξ_6.

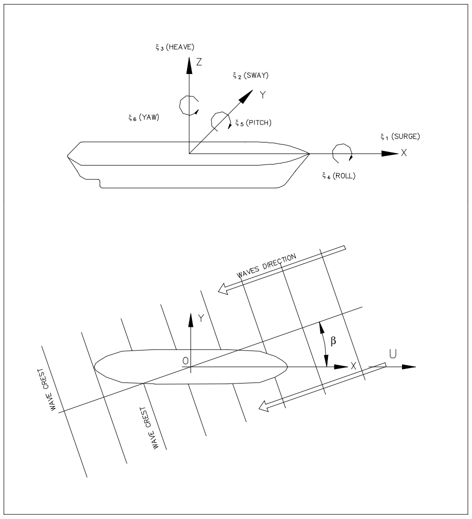

Fig. 1 - The right-hand tri-axial coordinate system and six modes of ship motion, and definition of the ship heading angle.

In general, the forces acting on the ship's hull consist of control forces from rudder, and active fins, environmental forces from wind and waves and reaction forces due to ship motions. In this study water is assumed incompressible, inviscid (although viscous effects are considered when roll damping is calculated), and deep.

When the problem of wave-induced parametric rolling is concerned, only the forces due to wave excitation and reaction forces due to wave-induced ship motions are taken into account. Other forces are assumed to be cancelled by each other, which means, the ship and relative course of the ship to the wave direction is kept constant during the time domain simulation for a given loading condition. Hence, the wave excitation forces consist of incident wave forces (or Froude-Krylov forces), diffraction forces, and the reaction forces of restoring forces and radiation forces. Surge motion, is assumed to be fixed.

In an approximate way then radiation and wave excitation forces are calculated at the equilibrium waterline using a standard strip theory, where the two-dimensional frequency-dependent coefficients of added mass and damping are computed by the Frank´s close fit method, and the sectional diffraction forces are evaluated using the Haskind-Newman relations [6].

A quasi-static approach is adopted to calculate the non-linear restoring coefficients in heave, roll, and pitch motions in waves, in which calculations are made over the instantaneous submerged hull. At this point it should be mentioned that a more sophisticated wave-induced parametric roll model could be adopted, where added masses and damping coefficients would be also calculated with consideration to the instantaneous submerged hull body under the wave surface. However, the hydrodynamic component of the parametric excitation is insignificant in comparison with quasi-hydrostatic excitation caused by the incident wave potential and the wave-induced heave and pitch motion.

The hydrodynamic component depends upon the overall submerged hull form, while the quasi-static hydrostatic component is strongly dependent upon wave passage and hull-shape (i.e. variations about the still waterline). For this reason and others associated with larger computational efforts only the quasi-hydrostatic component of the parametric excitation is taken into account at this stage.

In particular the roll added inertia and radiation coefficients are taken to be linearly proportional to the roll acceleration and velocity, respectively. Hence, the hydrodynamic memory effect due to roll motion and consequently its effect, expressed as roll damping, is practically negligible at frequencies lower than 0.5 [rad/s] (see Fig. 2).

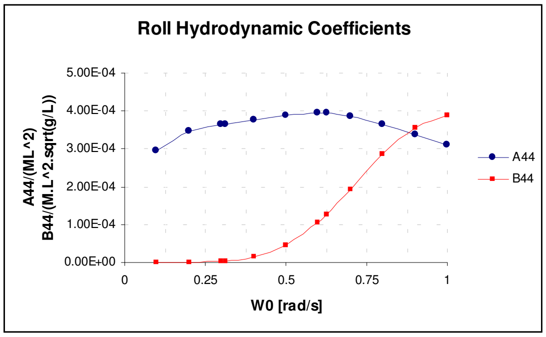

Fig. 2 – Added mass and damping coefficients of roll motion.

Under the assumptions presented before all hydrodynamic forces are linear, and combining these with the mass forces one obtains six linear coupled differential equations of motion, given by:

$$\sum_{j=1}^{6}\left\{\left(M_{kj}+A_{kj}\right)\ddot{\xi}_j+B_{kj}\dot{\xi}_j+C_{kj}(t)\xi_j\right\}=F_k \qquad \text{k, j} = 1,\dots,6 \qquad (2)$$

Here the subscripts k_j are associated with forces in the k-direction due to motions in the j-mode (k=1, 2, 3 represent the surge, sway and heave directions, and 4, 5, 6 represent roll, pitch and yaw directions). M_{kj} are the components of the mass matrix for the ship, A_{kj} and B_{kj} are the added mass and damping coefficients, $C_{kj}(t)$ are the hydrostatic (time dependent) restoring coefficients, and F_k are the complex amplitudes of the exciting forces, where the forces are given by the real part of $F_k e^{i\omega_e t}$.

If the ship travels along a prescribed path β at an initial steady velocity U (see Fig. 1), she will encounter the regular wave crests with a frequency of encounter, given by:

$$\omega_e = \omega - kU\cos\beta \qquad (3)$$

where the surface elevation of a regular wave is given by:

$$\eta_w = \eta_w^a \cos k\left[x\cos\beta - \left(c - U\cos\beta\right)t\right] \qquad (4)$$

In irregular seas, it is possible to describe the equations of motion given by the sum of sinusoidal waves yielding to an irregular wave profile given by:

$$\eta_w = \sum_{n=1}^{N}\eta_{w_n}^a \cos\left[\begin{array}{l}\dfrac{\omega_n^2}{g}x\cos\beta - \\ \left(\omega_n - \dfrac{\omega_n^2}{g}U\cos\beta\right)t + \varepsilon_n\end{array}\right] \qquad (5)$$

where N is the number of component waves, ω_n the circular frequency, ε_n the random phase angle, and $\eta_{w_n}^a$ the amplitude of the n-th component waves which are given by the wave spectrum $S(\omega)$.

Therefore, in this study parametrically excited roll response in regular and irregular head seas is investigated where linear ideal flow hydrodynamic added mass, damping coefficients and wave excitation forces are considered and non-linearities are introduced only via hydrostatic terms time dependence associated with wave passage and hull-shape, and non-linear roll damping. It has been found for example by Fonseca and Guedes Soares [7, 8] that this type of approach describes well non-linear motions.

3. ROLL DAMPING ASSESSMENT

The damping coefficient can be obtained from free decay experiments, in which the model is released from a given inclination angle to freely roll in calm water with no forward speed. Using adequate instrumentation a decay curve data points as shown in Fig. 3 were obtained.

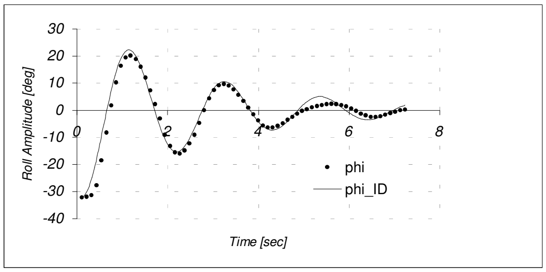

Fig. 3 – Ship model free decay and non-linear simulation curves.

Then using appropriate parametric identification techniques, the coefficients B_{441} and B_{442}, can be obtained by fitting equation 6 to the recorded free decay experiment data.

$$(M_{44} + A_{44})\ddot{\xi}_4 + B_{441}\dot{\xi}_4 + B_{442}\dot{\xi}_4\left|\dot{\xi}_4\right| + C_{44}\xi_4 = 0 \qquad (6)$$

where B_{441} is the linear damping coefficient, and B_{442} is the quadratic damping coefficient. Because of the limited number of cycles of roll decay traces, the energy balance method is adopted. This method is based on the concept that the rate of change of the total energy in roll motion is equal to the rate of energy dissipated by the roll damping. As shown in equation 6, it is also assumed that the ship is under uncoupled roll motion during the free decay experiments. According to reference [9], the equivalent linearised roll-damping coefficient is therefore related to the dissipated energy and is given by:

$$b_{44_{eq}} = b_{441} + \frac{8}{3\pi} b_{442}\xi_4{}^a \omega_{44_n} \qquad (7)$$

Very good agreement is found between the 'identified' curve obtained from numerical integration, given in Fig. 3 by the continuous line, and the experimental data points acquired during free decay test.

From previous studies [3] it is known that when roll damping is tuned to model test results, a very good correlation of the roll motion can be achieved between model tests and the numerical analyses. Particularly, the magnitude of the roll response during parametric rolling is dictated in large part by the amount of viscous damping in the roll degree of freedom. To account for these damping effects in this study, it was decided to compare the empirical roll damping established from the free-decay model tests with an applicable analytical method. Hence, it was found that the roll-damping method of components, presented in reference [10], could also be used to accurately calculate the total roll-damping coefficient at zero advance speed.

4. INSTANTANEOUS NON-LINEAR RESTORING FORCES AND MOMENTS

Instantaneous restoring forces and moments are calculated from the exact determination of the ship's displacement and its centre of buoyancy at each step. The relationship between the wave surface and the ship's hull in the seaway was established taking into account the responses of the ship in the five degrees-of-freedom as well as the ship speed and heading. In fact, in this condition of excitation of motion the ship basically has no response in sway and yaw modes but the computer code allows for it.

As demonstrated in reference [3], considering a monohull in longitudinal regular waves about the same length as the ship' s length variations of instantaneous restoring forces and moments are due to sectional beam variations dy/dx, which depend on ship' s hull vertical flare dy/dz (see Fig 4).

Hydrostatic forces and moments calculations are made using the pressure integration technique over the ship hull, rather than using area and volume integration of the ship offsets. The original theoretical approach to the pressure integration technique outlined by reference [11] has been adopted in conjunction with a practical method to generate the panels required to calculate the hydrostatic pressure

distribution under either a regular or irregular wave profile [12].
Therefore, in this model non-linearities are considered in the heave, roll, and pitch restoring terms, taking into account all the instantaneous variation of the hull-shape in waves including eventually the occurrence of deck submergence.

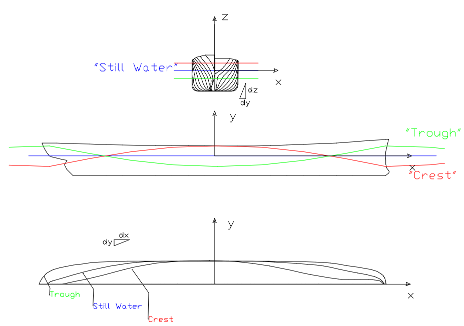

Fig. 4 – Variation of sectional beam on longitudinal waves.

5. EXPERIMENTS IN REGULAR HEAD SEAS

Tests were conducted at the small towing tank of the UCL [13], equipped with a flap type wavemaker and a towing carriage. A typical fast form of a large refrigerated container ship was selected as a subject of the experimental studies on parametric rolling. The body lines are presented in Fig. 5. The model was of wooden and GRP construction, the hull is vertical sided amidships above the waterline and completely unappended and unpropelled. The principal particulars of the model are given in Table 1. The model was constrained in yaw, sway, and surge.

This 1:100 scaled model was instrumented so that traces of its dynamic response in waves were produced. In the experiments with no advance speed, the ship's model loading condition was maintained and set to be compliant with the applicable IMO intact stability criteria in force at that time [14]. During these runs, wave amplitude and frequency were varied in order to investigate the sensitivity of the scaled model to parametric resonance during which capsize can be induced.

Then in order to obtain parametric resonance with advance speed, the roll stiffness of the model was adjusted. This variation on transverse metacentric height was achieved displacing vertically the existing masses on the stud mast. During the run along the tank with regular waves of wavelength about the model' s length, low cycle resonance was obtained by varying the forward speed of the carriage.

Roll decrement tests were performed for different initial angles, and inclining experiments were conducted to assess damping and transverse stability characteristics, respectively.

It was found that rolling was induced only in a few conditions. As shown in Fig. 7.a (scaled by a factor 100:1), the most pronounced rolling occurred when the model was adjusted to a roll period twice the period of the wave.

As can be seen from Fig. 7.a, the roll angle increases on each successive swing up to defined limited amplitude, and is at half the wave frequency. In Fig. 7.b, it can be seen that the pitch motion was regular and of the same frequency as the waves. Heaving motion was also regular and had the same periodicity of the waves. The sway and yaw motions were very small and irregular.

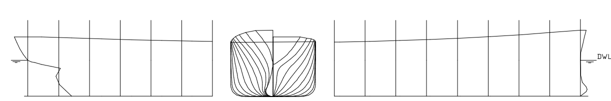

Fig. 5 – Body lines of the refrigerated container ship model.

Table 1 - Refrigerated container ship model particulars.

L_{BP}	= 128.0 cm	C_M	= 0.973
L_{OA}	= 132.4 cm	C_P	= 0.578
B_{WL}	= 18.1 cm	C_W	= 0.731
D	= 11.5 cm	S_W	= 3019 cm^2
T_{WL}	= 7.68 cm	T_n	= 2.0 sec
W	= 10.07 Kg	GM_t	= 0.17 cm
		Scale	= 1:100

6. NUMERICAL RESULTS

As mentioned, the strip theory computer program was used to perform the calculation of the added mass and damping coefficients, and wave excitation forces and moments for a given wave encounter frequency at the equilibrium waterline. With respect to roll damping coefficient, viscous components were then added to wave-making component in order to obtain the total roll-damping coefficient. These are fed into equations 2, which are then integrated to simulate a non-linear time-domain wave-induced parametric resonance condition. For this purpose the computer code presented herein was developed to incorporate ship motions calculations, instantaneous restoring forces and moments calculations, and then to perform a numerical integration of the equations using a 4th order Runge-Kutta algorithm on a step-by-step basis.

Numerical results of motions excited by longitudinal regular waves obtained from simulations showed that sway and yaw were insignificant in comparison with heave, roll, and pitch responses. The pitch and roll responses were then used for comparison with time series recorded in the experimental programme for the model.

6.1. Simulated instability in regular head seas

Initially, several numerical simulations were carried out at the same load condition utilised for the UCL model tests for a number of combinations of speeds and wave conditions.

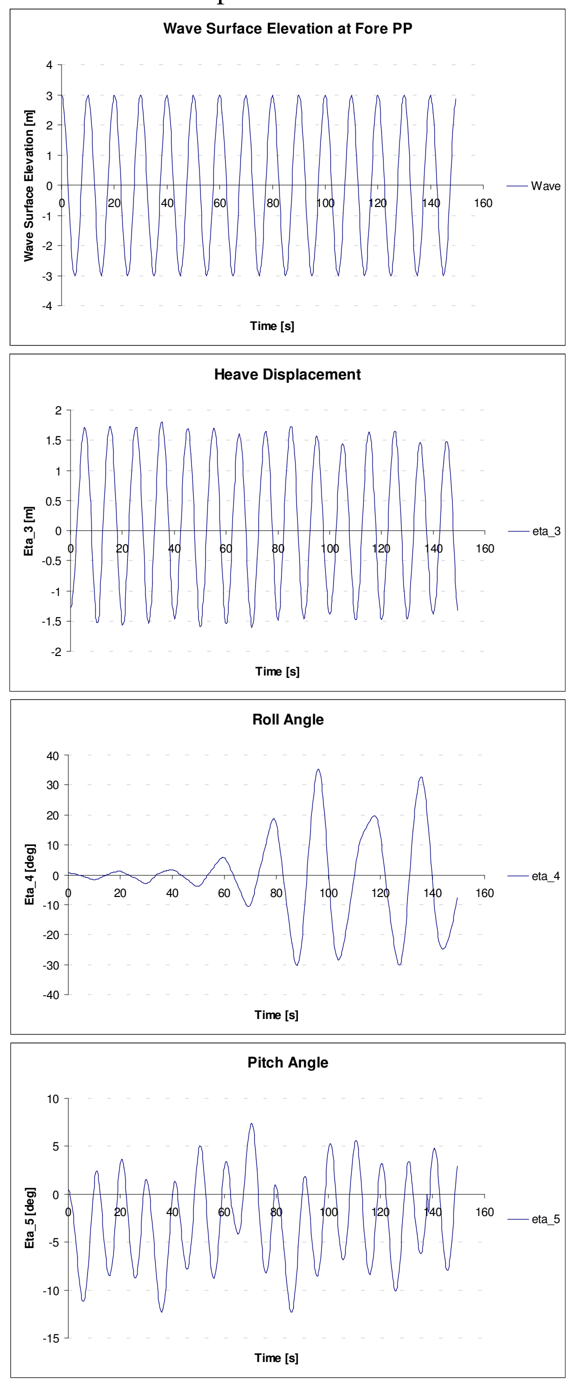

Fig. 6 - Numerical simulation in regular head seas (H_W = 6 [m] and T_W = 14 [s]).

Then, as shown in Fig. 6 the same wave conditions (corresponding to the most critical wave-induced parametric rolling condition) were used in the simulations as in the model tests. Plotted are wave surface elevation, and heave, roll, and pitch displacements during low-cycle parametric resonance.

In Fig. 7.a wave surface elevation and ship's roll motion records from simulation and experiments are compared, and, obviously the parameters involved in the equations are equivalent to those used in model calculations. In what concerns longitudinal plane motions good agreement is again obtained between numerical simulations and experimental results, as compared in Fig. 7.b for pitch motion.

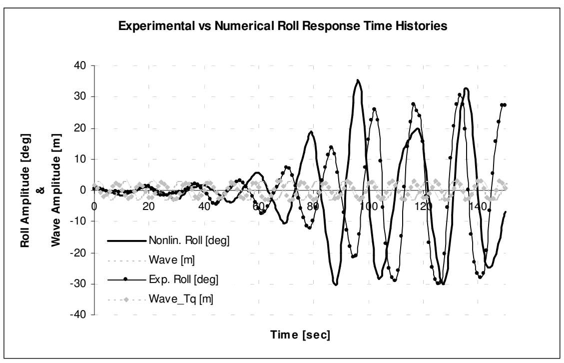

Fig. 7.a - Comparison of numerical integration with UCL experimental results in regular waves – roll motion.

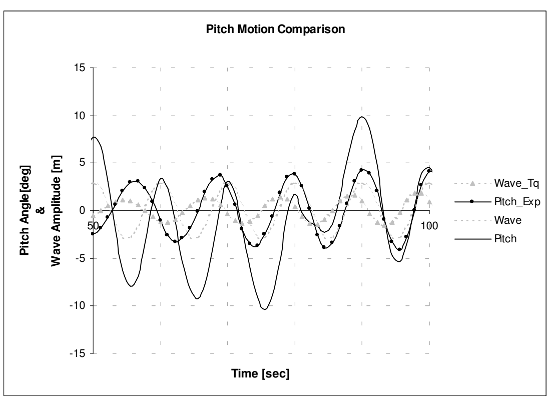

Fig. 7.b - Comparison of numerical integration with UCL experimental results in regular waves – pitch motion.

It can be seen that the main features of parametric roll phenomenon are predicted by the computer programme. Like the model test results, there is a period with no rolling, and then roll angle increases on each successive swing up to defined limited amplitude, and is at half the wave frequency. Also, it can be seen from Fig.'s 6 and 7 that roll angles increased from a few degrees to over 30º in only five roll cycles, and at this stage that the model was pitching to angles of about 5º, respectively. Moreover, there are two pitch cycles for each roll cycle, and the model is always pitched down by the bow at maximum roll.
A question of particular interest in non-linear systems is the existence of closed trajectories, as such trajectories imply periodic motion, and eventually a stable state known as the limit cycle. As shown in Fig. 8 for the wave-induced parametric rolling condition the response is practically sinusoidal and the phase plane plot is nearly an elliptical spiral.

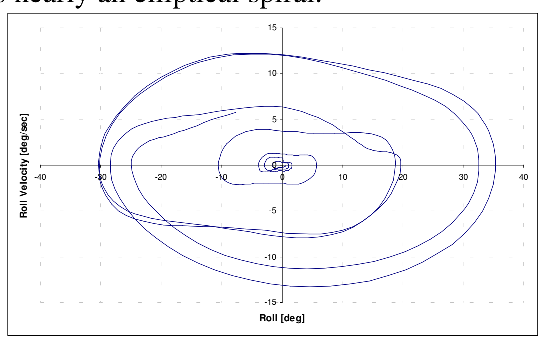

Fig. 8 – Roll phase diagram in regular waves.

As shown in a previous study [3] with a linear formulation the roll motion will not have a limit cycle behaviour. The non-linear model proposed here captures the time variation of the roll restoring moment, which is the most important effect that contributes parametric resonance at the initial transient stage. It also takes into consideration the effect of deck submergence on restoring moments at the steady state stage, which prevents roll angles from building up to infinity.

6.2. Simulated instability in irregular head seas

For the simulations in irregular waves, both randomly distributed amplitude and phase components are utilised to generate an incident wave realization. More specifically, the modelling of the incident wave spectrum is made by a finite number of harmonic waves where a lower and an upper limit for the wave frequency, $\omega_{\min}$ and ω_{Max} are defined. The continuous incident wave spectrum is therefore discretised by a number of harmonic wave components N_{wc} of frequency and amplitude:

$$\omega_n = \omega_{\min} + n_{seed}.\Delta\omega \tag{8}$$

$$\eta^a_{wn} = \sqrt{2.S(\omega_n).\Delta\omega} \tag{9}$$

where:

$$\Delta\omega = \frac{\omega_{Max} - \omega_{\min}}{N_{wc}} \tag{10}$$

The phase angles of the regular waves ε_n are also randomly distributed in the entire range $\Delta\varepsilon$ = [0, 2π], and is given by:

$$\varepsilon_n = n_{seed}.\Delta\varepsilon \tag{11}$$

Although, the wave energy of the discretised wave systems resulting from the above approach equals the wave energy of the incident irregular seaway adopted, this wave realization in space is strongly dependent upon the seed number n_{seed} provided as input for generation of random waves.

In this case, the incident wave is described by a JONSWAP spectrum $S_{JS}(\omega)$, with a peak enhancement factor γ = 3.33, a significant wave height H_s = 6 [m], and a peak period T_p = 14 [s] (see Fig. 9).

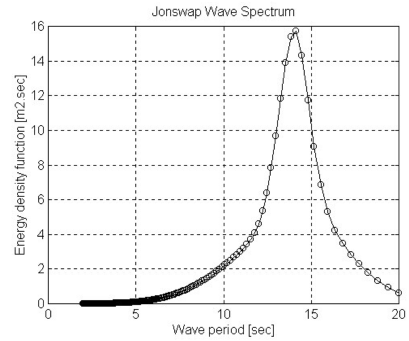

Fig. 9 – JONSWAP spectrum (γ = 3.33, H_s = 6 [m], and T_p = 10 [s]).

$$S_{JS}(\omega) = K.S_{PM}(\omega).\phi_{JS}(\omega) \tag{12}$$

where:

- $S_{PM}(\omega) = \frac{4\pi^3 H_S^{\,2}}{T_Z^4}\frac{1}{\omega^5}\exp\left(-\frac{16\pi^3}{T_Z^4}\frac{1}{\omega^4}\right)$ is the energy density function of the Pierson-Moskowitz spectrum;
- $\phi_{JS}(\omega) = \left(1 - e^{-1.25}.\ln\gamma\right)\gamma^{\exp\left[-\frac{1}{2}\left(\frac{\omega_p - \omega}{\sigma\omega_p}\right)^2\right]}$ is the function of the JONSWAP spectrum;
- K is a factor such as $K = \int_0^\infty S_{PM}(\omega)d\omega = \frac{H_S^{\,2}}{16}$.

The wave spectrum is approximated by a wave system consisting of 41 elementary waves, and as far as the reproduction of aperiodicity for the modelled seaway is concerned, this is clearly achieved (see Fig. 10).

The simulation of ship motions is again performed for a ship heading of 180º and a zero advance speed condition. Therefore added mass and damping hydrodynamic coefficients are calculated for the considered values of peak spectral frequency and significant wave height. Since parametric rolling is a resonant phenomenon, it was found that contrarily to the regular wave's scenario in irregular seas ships might not be prone to regularly exhibit parametric rolling due to waves groupiness effect. More specifically, in irregular seas the required synchronisation between waves and roll response period might not be sustained long enough due to variations of the wave realization in time and space.

In Fig. 10, the simulation records of wave surface elevation, heave, roll, and pitch vessel responses in the auto-parametric rolling scenario are presented. Fig. 11, shows a combined view of the roll and pitch responses in irregular waves, where roll angles exceeding 30º to each side, after 130 seconds are evident. Comparisons of motion time histories between the present predictions and model tests are not possible because free-running experiments in irregular waves have not yet been carried out.

However, the coupling between maximum roll and bow down pitch observed in the regular waves model tests is also detected in irregular waves.

In Fig. 12 the same result is presented in the form of roll phase diagram.

As in regular waves, the roll motion builds to large amplitudes and when either the wave period is changed or the wave height diminishes, the parametric resonance disappears. Therefore, parametric rolling can occur in irregular seas conditions provided again there is sufficient encountered energy near twice the natural roll period.

Although computer code does not account for all the hydrodynamic phenomena, the code is capable of predicting parametric rolling in head seas and can be used to predict if a vessel is prone to exhibit parametric rolling in an early design stage.

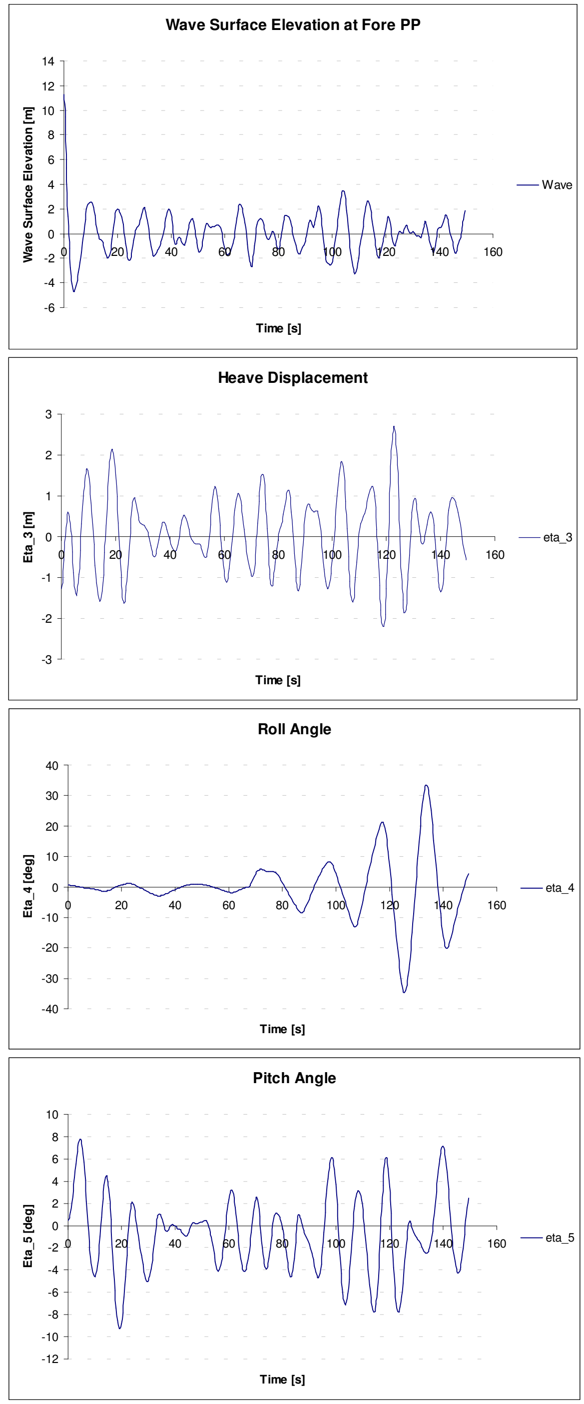

Fig. 10 – Numerical simulations in irregular waves.

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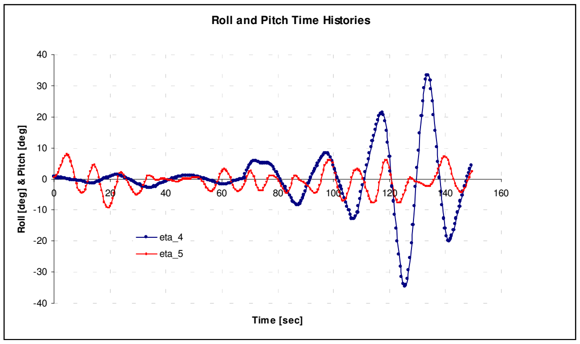

Fig. 11 – Combined view of roll and pitch motions in irregular waves.

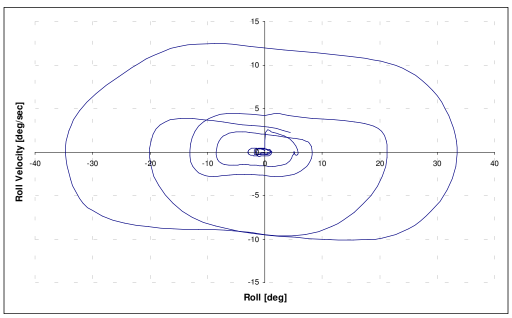

Fig. 12 – Roll phase diagram in irregular waves.

7. CONCLUSIONS

In this study a non-linear numerical model is utilised to simulate parametric resonance in both regular and irregular waves. In addition, special attention has been given to the usefulness of model experiments conducted under conditions as realistic as possible to validate the theoretical approach proposed.
The main concluding remarks are summarised as follows:
a) It has been confirmed that a statically stable ship encountering waves of her own length and a frequency twice her natural roll frequency will experience a wave-induced parametric rolling situation. Therefore, under these conditions, roll angles exceeding 30º to each side can be produced rapidly, resulting sometimes in cargo losses and ship's damage, or eventually in capsize.
b) When roll damping is tuned to model test results, a very good correlation of the roll motion can be achieved between model tests and the numerical analyses. Therefore, in this study, the linear and quadratic damping moments are linearised by means of the energy balance method and expressed by an equivalent linear roll-damping coefficient, which compares well with the analytical method of components.
c) A non-linear numerical model of parametric resonance taking into consideration deck submergence and other non-linearities on restoring forces and moments of ships at sea is then proposed. Good agreement is found between the time domain simulation of ship's responses in a longitudinal seaway and experimental data. Moreover, only representing coupled heave, roll, and pitch responses in long-crested irregular waves parametric rolling can be properly simulated.
d) In irregular waves the occurrence of parametric rolling can be highly dependent on the input seed number and a large number of simulations can be required to obtain a parametric resonance condition with roll angles exceeding 30º.

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